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High-Field Behavior in the Kondo Effect*

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Third-order perturbation-theory calculations of the conduction-electron scattering in dilute magnetic alloys are examined in the regime $\mu_B H \gg kT_K$, T_K the Kondo temperature. The results are compared with the high-field resistivity of dilute *Cu*Cr alloys, with an *S*-matrix calculation of the field dependence of the Hall coefficient, and with the high-field behavior of the thermopower in *Au*Fe alloys. Suggestions are made for systematic studies in the high-field regime.

It has been pointed out that perturbation theory for the s-d exchange Hamiltonian converges in two regimes: for high temperatures, $T > T_K$, T_K the Kondo temperature, and also for high fields, $\mu_B H > kT_K$.[1] In this paper we shall investigate the properties of perturbative calculations of the transport properties of dilute magnetic alloy systems in the high-field regime. A third-order perturbation-theory calculation, whose details have been published elsewhere,[2] for the resistivity ρ will be compared with the high-field low-temperature measurements by Daybell and Steyert[3] in *Cu*Cr, and a similar calculation of the Hall coefficient[4] R will be compared with More's[5] results in an *S*-matrix theory valid at all values of T and H. The high-field behavior of the thermopower[6] will also be discussed.

An energy average with a weight factor given by the derivative of the Fermi function is required in

obtaining ρ and R from the electron lifetimes. In zero field this averaging introduces roughly 20% effects in the temperature dependence,[7] but in high fields the region of rapid energy dependence in the lifetimes is far removed from the Fermi surface, where the weight factor is significant, and we shall therefore ignore the energy dependence of the lifetimes. We then have

$$\rho = (3\pi^2\hbar^3 m/e^2 p_F^3)\ (\tau_+ + \tau_-)^{-1}\ , \tag{1a}$$

$$R = 2R_0(\tau_+^2 + \tau_-^2)/(\tau_+ + \tau_-)^2\ , \tag{1b}$$

R_0 is the zero-field Hall coefficient, and $\tau_\pm$ are the up- and down-spin lifetimes evaluated at the Fermi energy ϵ_F. In third-order perturbation theory these lifetimes are[2,4]

$$1/\tau_\pm = (k_F m/\pi\hbar^3)\ v_0 c\ \{V^2 + J^2[1 + 4JG(H, T)] \times [S(S+1) - \langle S_z\rangle \tanh(\mu_B H/kT)] \mp 2VJ\langle S_z\rangle\ [1 + 2JG(H, T)]\}\ , \tag{2}$$

$$G(H, T) = \frac{3z}{2\epsilon_F}\ \left[\,\cdot + \tfrac{1}{2}\ln\frac{kT}{4\epsilon_F} + \tfrac{1}{2}\ I\!\left(\frac{\cdot_B H}{kT}\right)\right], \tag{3}$$

$$I\ (x) = \tfrac{1}{4}\int_0^\infty dx'\ \frac{\ln|x'^2 - x^2|}{\cosh^2 \tfrac{1}{2}x'}\ . \tag{4}$$

We have assumed the impurity g value is 2; V and J are the ordinary and exchange potentials in the s-d Hamiltonian ($J < 0$ gives antiferromagnetic exchange); v_0 is the atomic volume, c is the impurity concentration, and z is the number of conduction electrons per atom. The behavior of $I(x)$ for large x is

$$I(x) \sim \ln x - \pi^2/12x^2,\quad x \geq 10. \tag{5}$$

For $\mu_B H \gg kT$ we have, making the approximation $V \gg |J|$ and keeping only J^3 terms,

$$\rho(H, T) = \frac{3\pi}{2\epsilon_F}\ \frac{m}{e^2\hbar}\ cv_0 \times \left\{V^2 + J^2\left[1 + \frac{6Jz}{\epsilon_F}\left(1 + \tfrac{1}{2}\ln\frac{\mu_B H}{4\epsilon_F}\right)\right] \times [S(S+1) - 4\langle S_z^2\rangle - \langle S_z\rangle]\right\}\ , \tag{6}$$

$$\frac{R}{R_0} = 1 + 4\left(\frac{J}{V}\right)^2\ \langle S_z\rangle^2\ \left[1 + \frac{6Jz}{\epsilon_F}\ \left(1 + \tfrac{1}{2}\ln\frac{\mu_B H}{4\epsilon_F}\right)\right]\ , \tag{7}$$

where there is implicit field and temperature dependence in $\langle S_z\rangle$. We see that in this high-field regime one obtains $\ln H$ terms replacing the $\ln T$ terms typical of the zero-field behavior of these Kondo effect systems. For antiferromagnetic J these $\ln H$ terms will cause the contribution of the exchange potential J to both ρ and R to decrease in magnitude as H is increased. However, if we use the spin polarization $\langle S_z\rangle$ obtained from free spins, which is saturated for $\mu_B H \gg kT$, we find that the spin factor in the J^2 part of Eq. (6) is $-3S^2$ and hence $\rho(H)$ will actually increase as a function of $\ln H$.

The experimental results obtained in *Cu*Cr[3] alloys show that in the range 6.94 kOe $\leq H \leq$ 25.8 kOe and $T \leq 0.2$ °K, $\rho(H)$ *decreases* proportionally with $\ln H$, the constant of proportionality being approximately 0.26 nΩ cm/ppm. This is in contradistinction to Eq. (6), assuming free-spin behavior of the spin polarization. However, incremental susceptibility measurements at the fields and temperatures of these resistivity measurements[8] show that the spin on the impurity is not saturated, and it is possible that $S(S+1) - 4\langle S_z\rangle^2 - \langle S_z\rangle > 0$ if we use the experimental spin polarization. However, the incremental susceptibility, proportional to $\partial\langle S_z\rangle/\partial H$, is not zero at a field of 21.4 kOe, and it is surprising that there is no additional field dependence in ρ due to the field dependence of $\langle S_z\rangle$. We should note that the incremental susceptibility is temperature independent in this temperature range, which, taken together with Eq. (6), explains the temperature independence, at fixed field, of the high-field resistivity in these *Cu*Cr alloys.

One possible explanation of this disagreement between our calculation and experiment in the sign of the $\ln H$ part of the resistivity is that the observed decrease is not due to the explicit $\ln H$ in Eq. (6), but rather to the increase of $\langle S_z\rangle$ in $S(S+1) - 4\langle S_z\rangle^2 - \langle S_z\rangle$, and that at still higher fields than those used in Ref. 3, where the impurity magnetization will be saturated, the experimental low-temperature plateau might increase with $\ln H$ in agreement with Eq. (6). This latter behavior is expected from the S-matrix calculation,[1] which shows that for $T \ll T_K$ the resistance *decreases* and then *increases* with the field. However, this S-matrix calculation does not successfully explain the experimental results since it does not show the observed low-temperature plateau for fixed field.

The high-field behavior of the Hall coefficient as given by Eq. (7) is that R will decrease logarithmically with H if we assume that $\langle S_z\rangle$ is independent of H for $\mu_B H \gg kT_K$. Similar behavior is seen in the lowest-temperature curve obtained in More's S-matrix calculation[5]: $T/T_K = 0.82 \times 10^{-2}$ for $\mu_B H > kT_K$. As has been noted elsewhere,[4] such a decrease must not be confused with a more important one due to orbital effects when $\omega_c\tau > 1$, ω_c the cyclotron frequency.[9] It thus appears that experiments to check the high-field $\ln H$ decrease in R will be much more difficult than those for the magnetoresistivity.

We should like to recall here that a similar calculation of the thermopower $S(H)$ in the high-field re-

gime[6] gave $1/H$ behavior in good agreement with experiments on AuFe.[10] This result could be physically understood by noting that the thermopower involves the derivative of the lifetime, proportional to $1/H$ in high fields, while the magnetoresistivity and the Hall coefficient involve the lifetimes themselves, giving $\ln H$ behavior.

We can conclude from this study that although the experiments would be difficult it would be very interesting to get systematic data in high fields and low temperatures (say, $\mu_B H/kT \sim 5, 10, 20$) for the magnetoresistivity, the Hall voltage and the thermopower, together with magnetization measurements. Indeed, while perturbation theory does show that $\ln H$ behavior will occur in the high-field regime for ρ and R, and $1/H$ behavior for S, there is considerable uncertainty about the meaning of the spin-polarization terms which appear for large H, and, specifically, one needs to know how much the impurity magnetization is saturated in these large fields.

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Low-Temperature Transport Properties of Dilute Silver-Manganese Alloys

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We report measurements of the electrical resistivity between 20 and 0.3 K and the thermal conductivity between 4 and 0.6 K of a number of dilute alloys of manganese dissolved in silver. The nominal concentration range of the alloys is from 0.005- to 1.1-at.% Mn. The electrical-resistivity curves for the more concentrated alloys show a resistance maximum at an antiferromagnetic ordering temperature T_{max} which is a characteristic for this alloy system. At lower temperatures, however, a resistivity that is approximately a linear function of temperature is observed. The Lorenz number for the alloys turns out to be concentration dependent, although the deviations from L_0, the Sommerfeld value for the Lorenz number, are generally not more than about 4%. The results are interpreted in terms of a phenomenological model based on the classical Kondo theory, together with a Lorentzian distribution of internal magnetic fields that exist at an impurity site.

I. INTRODUCTION

If certain magnetic impurities are dissolved in sufficient quantity in the noble metals, then at a low temperature a minimum followed by a maximum at lower temperatures is generally observed in the electrical resistivity of these alloys.[1-3] Well below the maximum, an experiment by McDonald[4] on Au-Fe seemed to indicate that the resistivity can become a linear function of temperature. This interesting observation made it desirable to us to make low-temperature transport measurements on other noble metals containing magnetic impurities. We report here measurements of the electrical resistivity between 0.3 and 20 K and the thermal conductivity between 0.6 and 4.2 K of a number of silver alloys containing manganese as impurities. Our results confirm the findings by other authors of a